

Finding zero's is an important part of sketching graphs.

Before we attack this question, we need one more tool:

Long Division:

how many times does 12 go into 12?
how many times does 12 go into 9?

5

$$\begin{array}{r} \boxed{12} \overline{) 1293} \\ \underline{-12} \\ 09 \\ \underline{-0} \\ 93 \\ \underline{-84} \\ \boxed{9} \\ \uparrow \\ \text{Remainder} \end{array}$$

$\boxed{12} \cdot 1$
 $\boxed{12} \cdot 0$

$$\begin{array}{r} 12 \\ \times 8 \\ \hline 96 \end{array} \leftarrow \text{too big.}$$

~~1293~~

$$\begin{array}{r} 12 \\ \times 7 \\ \hline 84 \end{array} \leftarrow \text{this works}$$
$$\begin{array}{r} 107 \\ \times 12 \\ \hline 214 \\ 1070 \\ \hline 1284 \end{array}$$

So: $\frac{107}{12} = 107 + \frac{9}{12}$

Polynomial Long Division

multiply $(x+2)$ by x^2 to match the leading term x^3

don't need to do anything to match leading term of 0

$$x+2 \cdot \overline{) x^3 + 2x^2 + 9x + 3}$$

multiply by 9 to match current leading term of $9x$

$$\underline{-(x^2 + 2x^2)} \quad \downarrow$$

$$\begin{array}{r} 0 + 9x \\ - \quad 0 \end{array} \quad \downarrow$$

$$9x + 3$$

$$\underline{-(9x + 18)}$$

$$\boxed{-15}$$

← Remainder

$$\begin{aligned} 3 - 18 &= -18 + 3 \\ &= -15 \end{aligned}$$

So:
$$\frac{x^3 + 2x^2 + 9x + 3}{x+2} = x^2 + 9 - \frac{15}{x+2}$$

Eg: Divide $6x^3 - 1$ by $x^2 + x + 1$

to keep things straight...

add in spaces for missing terms

think $\frac{6x^3}{x^2} = 6x$
 think $(x^2)(-6) = -6x^2$

$$\begin{array}{r}
 x^2+x+1 \overline{) 6x^3 + 0x^2 + 0x - 1} \\
 \underline{-(6x^3 + 6x^2 + 6x)}
 \end{array}$$

$$\begin{array}{r}
 -6x^2 - 6x - 1 \\
 \underline{-(-6x^2 - 6x - 6)}
 \end{array}$$

$$\boxed{-1 - (-6) = -1 + 6 = 5}$$

5

↑
remainder

10

So: $\frac{6x^3 - 1}{x^2 + x + 1} = 6x - 6 + \frac{5}{x^2 + x + 1}$